EXPLORATION OF SHAPE SIMILARITY MATCHING USING NURBS-WARPING

Kim Meng Liang[1], Mandava Rajeswari[2], Bee Ee Khoo[3]

School of Industrial Technology
University Science Malaysia
11800 Minden Penang
Malaysia
e-mail: liangkm@hotmail.com[1], mandava@usm.my[2], bekhoo@usm.my[3]

ABSTRACT: Large image databases are used in many multimedia applications such as entertainment, business, art, engineering, and science. Searching information in large images repositories is a problem of great importance for the development of visual information system. Traditionally, image databases have been accessed by textual queries, which require the indexing of the entire database content. This textual annotation is a time consuming procedure and may lead to inaccurate retrieval results. To overcome this difficulty, content-based image retrieval was proposed. Content-based image retrieval is a system in which images are indexed by their visual content, such as colour, texture and shape. Many researchers extensively discuss the retrieval systems based on shape. In this paper, we propose a new matching technique to determine the similarity between two shapes based on the Non-Uniform Rational B-Spline (NURBS)-Warping method. The proposed method is inspired by the manner in which humans perceive objects and compare them. In this method, the two main factors considered are: the warping candidate and the warping attraction force. In this approach, the query image is represented by NURBS, with position of the control points and their weights as the variable parameters. The query image is then warped by an attraction force to match the shape of the database image, by adjusting the above parameters. The Total Cumulative Change in the Position of the Control Points (TCCPCP) and the Degree of Matching (DOM) are used as a measure of the similarity between the query and the database images. In the warping process, gradient vector flow (GVF) field, which is developed from the database image itself, is used as the attraction force. The effectiveness of this NURBS-Warping method in shape-based retrieval is examined by carrying out experiments on a set of silhouette free form database images. The overall retrieval results in the experiments show that NURBS-Warping method is able to derive an accurate shape similarity measure and is a potential method for use in shape-based retrieval systems.

Keywords: NURBS, Shape-Based Image Retrieval, Shape Similarity Measure, Deformation

1. INTRODUCTION

With the advances in digital imagery, large accessible data storage, internet repositories, and image applications, information conveyed through images is gaining in importance. There is a need to find a desired image from a collection of images, which is shared by many groups including journalists, engineers, historians, designers, teachers, artists and advertising agencies. Due to the emergence of large scale image databases, traditional methods of indexing images through text annotation is becoming outdated. Therefore, an effective retrieval approach is needed, to retrieve images that are relevant to a query from a database.

Content-based image retrieval is one of the image retrieval method proposed (V.N.Gudivada & J.V.Raghavan, 1995). It relies on the characterization of primitive perceptual features such as colour, shape and texture that can be automatically extracted from the images themselves. Retrieval by shape is considered one of the most difficult and challenging aspects of content-based search (R.C.Veltkamp & M.Hagedoorn, 200). Many shape retrieval techniques have been developed and many systems, both research and commercial, have been built. Researchers have provided a variety solution for shape representation and measurement of the difference between any two shapes. Shape representation plays an important role in the shape-based retrieval system because shape similarity measurement techniques are related to the methods that are used to model the shapes. In general, shape-based retrieval systems are distinguished by the shape representation model. The two categories of shape representation are: feature-based approach and shape transformation approach (A. D. Bimbo, 1999). In feature-based approach, various features utilised are: global features technique (W.Niblack, 1993), moment invariants (M.K.Hu, 1962), 2-D fourier transform (Y.Rui, 1996), wavelet transform (S.Mallat, 1989), B-spline (F.S.Cohen et. al., 1995), turning function (E.Arkin et. al., 1991), circular autoregressive model (R.Kashyap & R.Chellapa, 1981), chain coding (H.Freeman & J.Saghri, 1961), curvature scale space (F.Mokhtarian & A.K.Mackworth, 1992) and medial axis transform (H.Blum, 1967). The details of various shape representation are found in (S.Loncaric, 1998). In these feature-based approaches, the commonly used similarity measure is the distance function in the multidimensional feature space. The most well known distance functions are Minkowski distance, Hausdorff distance and Frechet distance (R.C.Veltkamp & M.Hagedoorn, 2000). The retrieval computation time of feature-based method is short making it attractive to large image database retrieval. However, feature-based approach tends to be sensitive to occlusion and small variations in shape that will affect the retrieval results. For most features, there is no guarantee that the human notion of "similarity" corresponds to the topological similarity in the feature space. Furthermore, it is difficult to find a coherent set of features that can faithfully reflect all possible shapes. In addition, shapes usually require the extraction of a great number of features in order to be reliably modelled.

Shape transformation approaches model shape similarity in a manner that closely resembles human perception. These methods are typically more robust in order to cope with shape distortion, support a higher precision in retrieval among highly similar images and allow comparison with partially occluded shapes. However in these methods, the retrieval computation time is longer compared to the feature-based approach. Therefore, they are only suited to small size images databases and run time indexing. The shape transformation model has been discussed in pattern recognition literature since the early work by Widrow (B.Widrow, 1973), and successively developed in the well-known snake model by Kass (M.Kass et. al., 1988). In the shape transformation approach, similarity measure is computed based on local comparisons of the contours of shapes, which is called the elastic matching. This similarity measure is related to a physical energy that is obtained by measuring the amount of deformation needed to explain the differences in two contours. A cost function weights the similarity of matched points on the two curves on the basis of local properties of the points, such as the distance between them, the difference in the tangent or curvature of the contour at those points. A number of different cost functions that have been proposed for elastic matching are described in (R.Basri et. al., 1985). Currently, the shape transformation model for shape retrieval has been widely investigated based on three different approaches. These are the representation of the model, the energy function and the method of optimisation. Bimbo (A. D. Bimbo & P.Pala, 1997) has proposed an elastic template matching for the user sketch. The sketched template is modelled as a piecewise fourth order B-spline polynomial function that is parameterised with respect to arc length. In each interval of deformation, the B-spline control points are implicitly updated by recalculating the B-spline formulation through a gradient descent technique. The measures of the elastic deformation, which comprised of stretching and bending energy, and degree of matching achieved, are used to evaluate similarity rankings of the database images. The shape similarity measure is computed through a back-propagation neural network. Although this approach provides a similarity measure that closely resembles human ranking, it involves a very high computation time. High computation time is needed in the recalculation of the reconstructed B-spline boundary points in each interval of deformation and the shape similarity value determination from the back-propagation neural network. In this paper, we introduce a new similarity matching approach to improve the computation time needed to determine the shape similarity measure, which is explained in the following sections.

1.1 The Proposed Approach

In our work, we propose a different shape similarity retrieval method based on elastic template, which is related to the work by Bimbo (A. D. Bimbo & P.Pala, 1997). The aim of our work is to extend the concept of shape transformation model with an intention to improve its efficiency, speed and applicability in a shape similarity-based retrieval system suitable for highly similar databases. Our main contributions are as follows. First, we introduce NURBS descriptors, such as the control points and the corresponding weights to model the query image, in a manner that the query image is regarded as a deformable template. The NURBS-based template is deformed by an external force in order to improve its match with the database image. This process is referred to as NURBS-Warping method. In this method, NURBS is chosen as an accurate shape descriptor because it gives more freedom and flexibility to reflect all possible query shapes compared to the B-spline (K.M.Liang et. al., 2002). Second, the NURBS allows a higher compact representation due to its additional properties compare to the B-spline, such as the availability of the non-uniform knot vector and additional parameter, which is the weight. With the high compact representation, only a small amount of NURBS descriptors are needed to warp towards the boundary of the database image. In addition, in contrast to the work by Bimbo, the NURBS-Warping process updates the position of the NURBS control points explicitly without any recalculation of the NURBS boundary points in each interval of deformation. Therefore, the speed of the warping process is improved. Third, NURBS-based template will eliminate the difficulty in determining the weights associated with the smoothness and continuity constraint. Fourth, we choose the gradient vector flow (GVF) as an external force in order to provides an efficient convergence of the query image towards the database image's boundary. Fifth, two parameters, the Cumulative Change in the Position of the Control Points (CCPCP) and the Degree of Matching (DOM) are introduced to measure the amount of energy spent during the warping process. Subsequently, the similarity value is determined by these two parameters using a simple thresholding step, which has the ability to reduce the retrieval computation time. Furthermore, NURBS-Warping method is a real time retrieval process, which does not required any pre-processing of the database images.

This paper is organised as follows. In Section 2, an overview of the deformable model by NURBS descriptor is presented. Section 3 introduces the NURBS-Warping process stages and the determination of the shape similarity measure value in order to display the ranking of the retrieved database images. Section 4 presents the experiments to evaluate the effectiveness of the proposed method used in the shape-based retrieval systems. Section 5 concludes the approach and discusses the future directions of our research.

2. DEFORMABLE MODEL USING NURBS

A deformable model is described as an energy minimizing spline that is guided by external and internal forces. Generally, the internal forces are determined solely by the query shape and are usually defined in terms of first and second order derivatives. The purpose of the internal energy is to maintain the smoothness and continuity of the deformable model in the warping process. The query shape, as a deformable template, gives an elegant method to simulate an elastic material that dynamically conforms to local image features. The two main drawbacks of a typical deformable template are the representation of the template by a large number of coefficients and an explicit formulation of the smoothness constraint. Unfortunately, this typical deformable template suffers from slow convergence speed because of the large number of coefficients to optimise, as well as from difficulties in determining the weights associated with the smoothness constraints. Furthermore, a high order derivative of the discrete curve is not accurate in noisy environments. In order to obtain a more stable deformable template in addition to yielding faster convergence results, we have modelled the deformation template as a piecewise fourth order NURBS polynomial function. In the NURBS warping method, only a small number of control points are required to represent any complex shape (W.Ma & J.P.Kruth, 1994). During the warping process, these control points are adjusted to obtain optimum matching. In addition, the NURBS-based template has the ability to control the smoothness of the deformed template. This ability is inherited by the NURBS characteristic that the curve continuity is determined by its NURBS basis functions that are determined by the Cox-deBoor algorithm (L.Piegl & W.Tiller, 1997). Thus, NURBS-based template allows a compact shape representation and implicit formulation of the smoothness constraint is built into the model.

The external forces required in the warping process are computed from the database image itself. There are two difficulties associated with generation of these external forces in the deformable model. First, the deformable template must be initialised close to the database image's boundary or else it is likely converge to the wrong result. Second problem is that the deformable template has difficulties progressing into boundary concavities. A good choice of external force increases the capture range of its force and guides the template towards the desired boundary. In our method, the external force selected is the gradient vector flow (GVF), introduced by Xu and Prince (X.Chenyang & J.L.Prince, 1998). GVF is insensitive to initialisation and has the ability to move into boundary concavities. The details of the definition of the GVF are given in the Appendix A.

In a typical deformable model, the deformable template, which is parameterised according to arc length, $x(s)$, $s \in [0,1]$ is deformed until the optimum matching is obtained by minimizing its energy function, E. The energy function comprises of internal energy, $\mathrm{E}_{\mathrm{int}}$ and external energy, E_{ext} and is given by,

$$\begin{aligned} \mathrm{E} &= \mathrm{E}_{\mathrm{int}} + \mathrm{E}_{ext} \\ &= \int_0^1 \alpha\left[x'(s)\right]^2 + \beta\left[x''(s)\right]^2 + \mathrm{E}_{ext}\left[x(s)\right]ds \end{aligned} \tag{1}$$

where α and β are the weights for the smoothness constraint.

Amini et.al (A.A.Amini et. al., 1990) have proposed a variational framework to find the parameterised deformable template, $x(s)$ that minimises the energy function. In this variational framework, the minimization process must satisfy the following Euler equation,

$$\alpha\left[x'(s)\right]^2 - \beta\left[x''(s)\right]^2 - \nabla\mathrm{E}_{ext} = 0 \tag{2}$$

In order to obtain a solution to Eq. 2, the deformable template is made dynamic by treating x as function of time t as well as s, which is defined as follows.

$$x_t(s,t) = \alpha\left[x'(s,t)\right]^2 - \beta\left[x''(s,t)\right]^2 - \nabla\mathrm{E}_{ext} \tag{3}$$

The solution of Eq. 3 is determined by discretizing this equation and solving the discrete system iteratively, as follows.

$$x_t = (A + \gamma\,\mathrm{I})^{-1}\left(x_{t-1} - \mathrm{E}_{ext}(x_{t-1})\right) \tag{4}$$

where γ is a step size and $(A + \gamma I)$ is a pentadiagonal matrix, which are fixed throughout the warping process. This pentadiagonal matrix is a stiffness matrix, which is computed by the smoothness constraint weights, α and β. The details of the numerical solutions are explained in (M.Kass et. al., 1988).

In our deformable model, which is modelled by the NURBS descriptor and using the GVF, v, the corresponding dynamic deformable function is obtained by replacing the external force, $-\nabla\mathrm{E}_{ext}$ in Eq. 4 with v, yielding.

$$x_t = (A + \gamma\,\mathrm{I})^{-1}\left(x_{t-1} - v(x_{t-1})\right) \tag{5}$$

With NURBS representation, the smoothness of the deformable template is implicitly controlled. Therefore, the pentadiagonal matrix may be ignored. Thus, the NURBS-based template has eliminated the difficulty of determining the smoothness constraint weights. In each deformation iteration, new control points are automatically updated by using a simplified dynamic deformable model function.

$$c_t = c_{t-1} + \kappa\left(v(c_{t-1})\right) \tag{6}$$

where c is the control point generated from the deformable template and κ is the external force (GVF) weight. The optimum position of the control points is achieved through the gradient descent technique. Although the gradient descent technique has the risk of resulting in a local minimum, it is less computational intensive compared to the global minimum optimisation techniques such as graduated non-convexity algorithms and genetic algorithms. In this work, gradient descent technique is chosen owing to its fast computation time in the warping process, which is an important factor in the shape-based image retrieval system.

3. NURBS-WARPING METHOD

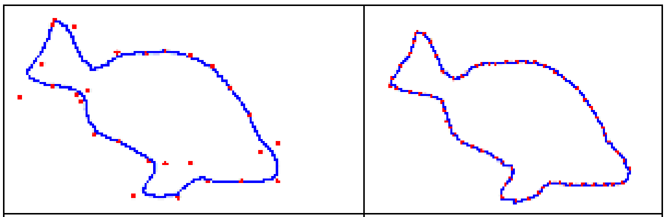
Although NURBS control points provide a compact and efficient deformable model, they usually are not adjusted explicitly owing to their location distribution, which is usually not on the curve itself. Therefore, Bimbo et.al (A. D. Bimbo & P.Pala, 1997) used the gradient descent technique to update the B-spline control points by minimizing an objective function. However, this minimization process needs to recalculate the reconstructed B-spline boundary point in each interval of deformation. On the other hand, Brigger (P.Brigger et. al., 2000) proposed a warping method by moving the B-spline node points rather than the control points. Although node points are located exactly on the curve, the computation process of the node points, which involves the determination of the knot value spacing, is still a complex problem to be solved. Thus, these methods generally required higher computation time, which is a disadvantage for a shape-based image retrieval. Motivated by these problems, we present the NURBS-Warping method. This method has three main stages: warping preparation, warping process and similarity ranking computation. The following sections present these stages and discussing the advantages of using NURBS descriptors in the warping process in order to determine the similarity between the query and the database images.

3.1 Warping Preparation

Generally, all shape transformation models require a normalization step in order to ensure the similarity measure parameters are solely determined from the warping process without being affected by the orientation differences between the query and database images. Therefore, before the warping process is carried out, the query image is normalized with a transformation matrix, (T) based on the orientation of the database image. NURBS property that the representation has the characteristic of shape invariance under affine transformation (L.Piegl & W.Tiller, 1997), which implies that the affine transformed curve is still a NURBS curve whose control points and weights are related to the original curve control points and weights through this transformation. Owing to this property, NURBS plays a significant role in the normalization step, in a manner that the NURBS descriptors need not be computed again for new normalized query image. Thus, the NURBS descriptors are normalized by subjecting them to the same transformation matrix, (T).

In order to achieve fast computation time, NURBS-Warping method adjusts the NURBS control points explicitly and does not involve any extra recalculation time in the reconstruction of NURBS boundary points during the warping process. To overcome the NURBS control points distribution problem discussed earlier, knot insertion method is introduced. Knot insertion method adjusts original NURBS control points and introduces additional NURBS control points without changing the representative query image's shape (L.Piegl & W.Tiller, 1997). This method allows the interpolated NURBS control points to be located nearer to the query image's boundary. With these interpolated control points on the boundary itself, the process of moving the control points is similar to the process of moving the boundary points of the query image itself, to match with the database image. The interpolated NURBS control points are used in the warping process to determine the shape similarity parameters. Fig. 1 shows the effect of the knot insertion method towards the distribution of the control points (red) around the database image's boundary (blue).

Then, the gradient vector flow field is generated from the edge map of the database image. In our implementation, silhouette images are used as database images. Therefore, the edge map in this context is referring to the boundary points of the database image itself. After the completion of the gradient vector flow fields computation, the interpolated NURBS control points are iteratively warped toward the database image's boundary under the attraction of this GVF field, using the gradient descent technique.



(a) (b)

Figure 1: (a) Control points distribution before knot insertion method (b) Control points distribution after knot insertion method

3.2 Warping Process

In the warping process, the NURBS-based deformable template is superimposed on the database image, and the interpolated NURBS control points are deformed using the specified algorithm based on Eq.5. In our method, the weights of the corresponding control points are fixed throughout the warping process in order to make this process simple and fast. The attraction force is stronger at the location, which is further away from the database image's boundary. The control points are adjusted until they reach the database image's boundary. The position of the control points will not change if all the deformed control points have converged and locked themselves at the boundary of the database image. The warping process is continued until the Cumulative Change in the Position of the Control Points value is equal to or less than a prespecified threshold value. Fig. 2 shows the position of the control points (red) being attracted to the database image's boundary (grey) in the GVF field after 5 iterations (one interval) of the warping process. It also shows that the deformed template (blue) at each interval has slowly deformed its shape until it is similar to the database image's shape.

In order to ensure that the similarity measure value reflects the effort spent throughout the warping process, the first similarity measure parameter is determined by the Total Cumulative Change in the Position of the Control Points (TCCPCP), which is sum of the CCPCP value at each interval until the completion of the warping process. In Fig. 3, the CCPCP value is computed by cumulating all the distance between the positions of the control points at the two different intervals by using Euclidean distance function. Thus, the TCCPCP value is implicitly representing the effort spent throughout the warping process.

Another similarity measure parameter is the Degree of Matching (DOM), which is computed at the end of the warping process. In this stage, two types of matching features are computed. The reconstructed NURBS boundary points, which are generated by the final deformed control points is superimposed on the database image's boundary. Then, the overlapping points are detected from the intersection between the reconstructed NURBS boundary and the database image's boundary. The first matching feature (FMF) is a measure of the closeness of the deformed query image towards the database image, while the second matching feature (SMF) is a measure of the closeness of the database image towards the deformed query image.

Mathematically, the FMF is the percentage ratio of the number overlapping points (NOP) and the number of database image's boundary points (NDI). The SMF is the percentage ratio of the number overlapping points (NOP) and the deformed query image's boundary points (NQI), as follows.

$$\mathrm{FMF} = \frac{\mathrm{NOP}}{\mathrm{NDI}} \mathrm{x}100$$

$$\mathrm{SMF} = \frac{\mathrm{NOP}}{\mathrm{NQI}} \mathrm{x}100$$

If only one of the above matching features is considered, the determination of the DOM value will be inaccurate because there is a risk that only a part of the deformed query image has completely covered the database image's boundary and vice versa. In our method, the DOM value is determined by the average value of the FMF and the SMF, as follows.

$$\mathrm{DOM} = \frac{1}{2}(\mathrm{FMF} + \mathrm{SMF})\mathrm{x}100 \qquad (7)$$

3.3 Similarity Ranking Computation

The Degree of Matching (DOM) and the Total Cumulative Change in the Position of the Control Points (TCCPCP) at the end of the warping process are used to derive a similarity measure value between the query image and the database image, and to display the ranking of the retrieved database images. Upon completion of the warping process for all the images in the database, these two similarity values parameters, which correspond to each image in the database, are filtered out in two simple thresholding steps. In the first step, the database images are filtered based on the DOM value with a desired threshold value. It is assumed that the most non-relevant database images are discarded out in this step. The most non-relevant images have low DOM value because the query image is hardly warped towards the database image's boundary and the query image's shape is far different from the database image's shape. In the second step, the accepted database images at the first step are filtered again based on the TCCPCP value with a desired threshold value. The aim of the second step is to keep the most relevant database images requested by the query image. After completion of the second step, the similarity ranking of the database images towards the query image are displayed.

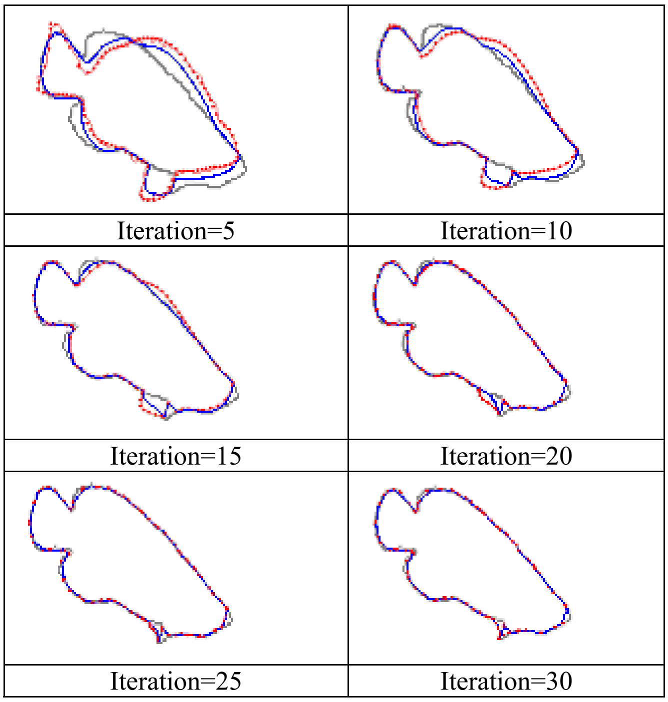


Figure 2: The control points of the query image are warped towards the boundary of the database image (grey) under the gradient vector flow field. The red boundary is computed at the previous 5 iterations and the blue boundary is computed at the recent iteration.

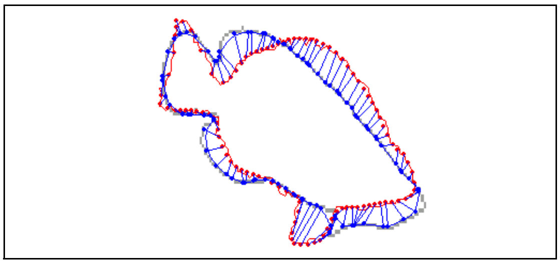


Figure 3: The cumulative change in the position of the control points is computed by adding all the distance of the joining lines (blue)

4. EXPERIMENTAL RESULTS AND DISCUSSIONS

In order to validate the effectiveness of the proposed method in the shape-based retrieval system, a set of silhouette database images, which contain 50 fish images and 50 tools images are used. In the experiment, 8 query images from the database are selected as query images of which 4 are the fish images and 4 are the tool images. Fig. 4 shows the 8 selected query images. The experiments in this work are aimed at proving that the NURBS-Warping method is able to retrieves images from the same group and is similar to the requested query image. The relevant images are retrieved based on the Degree of Matching (DOM) and the Total Cumulative Change in the Position of the Control Points (TCCPCP). The most relevant images have the highest DOM value and lowest TCCPCP value. The database images are filtered based on two DOM threshold value and TCCPCP threshold value, which are 65 and 700 respectively. Fig. 5 shows only the top five ranking images retrieved from the database.

In order to validate that the NURBS-Warping method is able to retrieves images from the same group to the requested query images, a percentage ratio of the number of the retrieved images, which are the same group with the requested query image and the total number of the retrieved images is computed. The results of this ratio are shown in Table 1.

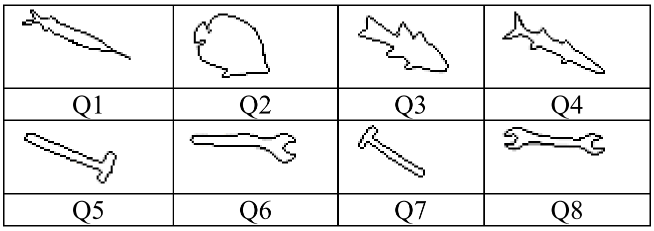


Figure 4: Eight selected query images are used in the retrieval system

Based on the results in Table 1, it shows that the NURBS-Warping method is a potential method in retrieving images from the same group requested by the query image. This is shown from the percentage ratio SI/IR results of Q2, Q3, Q5, Q6, Q7, Q8 and Q9. For the tool query images, the relevant images are mostly retrieved from the tool database, which shows that the NURBS-Warping method is capable of retrieving images accurately from a highly similar database. The percentage ratio of Q1 and Q4 are lower than 50% because the particular fish shapes itself are very similar to tool shapes, which both shapes have long and thin structure.

From the result shown in Fig. 5, all the query images have accurately retrieved three highest ranked database images. For the fish query images, retrieval results of Q2 and Q3 are the most accurate in their retrieval. This happens because these two shapes of the fish query images are very unique in the fish and tool database. For the tool query images, the best retrieval result is shown by Q8 in which all the retrieved results with top five ranking are very similar to Q8. The shape of Q8 is unique compared to other images in the tool database, which not only has long thin body but also with two heads at both ends. For other tool query images such as Q5, Q6 and Q7, the retrieval results are also considered similar to the requested query images. This similarity is shown by the same structures of the retrieved images' shape, which are having a long thin body with a head at the end. Based on the overall retrieval results shown in Fig. 5, it may be concluded that NURBS-Warping technique is an effective method to retrieve images from a highly similar database.

5. CONCLUSIONS AND FUTURE WORK

From the results and discussions presented in Section 4, it is clear that NURBS-Warping method is a powerful and potential shape similarity matching technique for use in shape-based image retrieval systems. We are presently working on the further development of this warping technique in order to improve the reliability and accuracy of the retrieval results from a larger set of database, which contains images that are highly similar to each other.

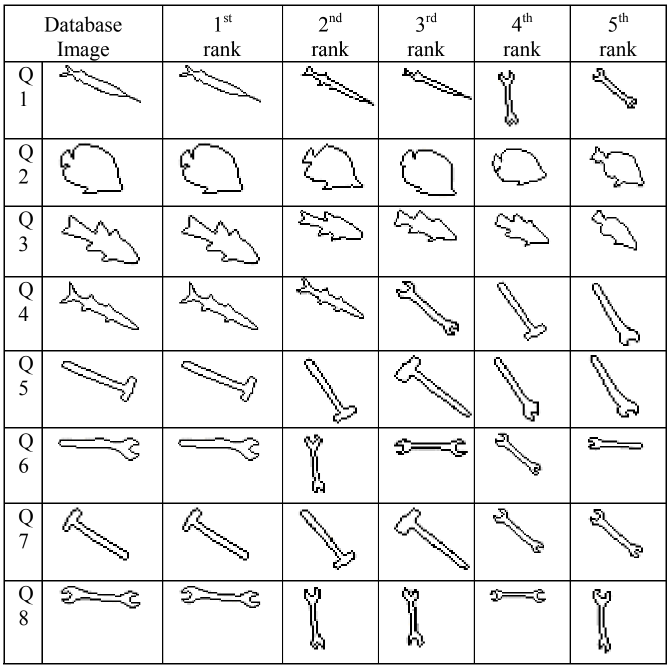


Figure 5: The results of the relevant database images retrieved by the 8 selected query images

Table 1: The percentage ratio of SI/IR (%) shows the effectiveness of the method in retrieving images that are relevant to the requested query images from the same group.

Query Image	No. of the same group image retrieved , SI	Total no. of images retrieved, IR	Percentage ratio of SI/IR (%)
Q1	5	33	15.15
Q2	5	5	100
Q3	10	11	90.91
Q4	5	15	33.33
Q5	29	35	82.86
Q6	35	39	89.74
Q7	26	31	83.87
Q8	40	46	86.96

APPENDIX A

Gradient vector flow is originated from the drawbacks of using the gradient magnitude as an external force. This external force is computed as a diffusion of the gradient vectors of a gray-level or binary edge map derived from the database image, without blurring the edges. Xu and Prince in [XChen98], define the gradient vector flow (GVF) field to be the vector field $v(x,y)=[u(x,y),w(x,y)]$, which is updated with every iteration of diffusion equations as follows.

$$u_{x,y}^{n+1}=\left(1-b_{x,y}\right)u_{x,y}^{n}+\left(u_{x+1,y}^{n}+u_{x,y+1}^{n}+u_{x-1,y}^{n}+u_{x,y-1}^{n}-4u_{x,y}^{n}\right)+c_{x,y}^{1}$$

$$w_{x,y}^{n+1}=\left(1-b_{x,y}\right)w_{x,y}^{n}+\left(w_{x+1,y}^{n}+w_{x,y+1}^{n}+w_{x-1,y}^{n}+w_{x,y-1}^{n}-4w_{x,y}^{n}\right)+c_{x,y}^{2}$$

where $b_{x,y}=f_x{}^2(x,y)+f_y{}^2(x,y)$

$$c_{x,y}^{1}=b_{x,y}f_x(x,y)$$

$$c_{x,y}^{2}=b_{x,y}f_y(x,y)$$

In this method, the initial values of u and w are the gradient values. A simple central differences is used to calculate f_x and f_y. The coefficients $b_{x,y}$, $c^1_{x,y}$ and $c^2_{x,y}$ are computed and fixed for the entire warping process. The details of the GVF generation are found in (X.Chenyang & J.L.Prince, 1998).

REFERENCE

A.A.Amini, T.E.Weymouth, R.C.Jain, Using dynamic programming for solving variational problems in vision. IEEE Trans. PAMI, 12(9):855-867 (1990)

B.Widrow, The rubber mask technique-II - pattern storage and recognition. Pattern Recognition, 5(3):198-211 (1973)

A. D. Bimbo & P.Pala, Image retrieval by elastic matching of user sketches. IEEE Trans. PAMI, 19(2):121-132 (1997)

A. D. Bimbo, Visual Information Retrieval. Morgan Kaufmann Publishers (1999)

E.Arkin, P. Chew, D.Huttenlocher, K.Kedem, J.Mitchel, An efficiently computable metric for comparing polygonal shapes. IEEE Trans. PAMI, 13(3):209-215 (1991)

F.Mokhtarian & A.K.Mackworth, A theory of multiscale, curvature-based shape representation for planar curves. IEEE Trans. PAMI, 14(8):789-805 (1992)

F.S.Cohen, Z.Huang, Z.Yang, Invariant matching and identification of curves using B-splines curve representation. IEEE Trans. Image Processing, 4(1):1-10 (1995)

H.Blum, A transformation for extracting new descriptors of shape. In Models for the Perception of Speech and Visual Forms, MIT Press, Cambridge, MA, pp.362-380 (1967)

H.Freeman & J.Saghri, On the encoding of arbitrary geometric configurations. IRE Trans. Electronic Computers, 10(2):260-268 (1961)

K.M.Liang, M.Rajeswari, B.E.Khoo, Free form shape representation using NURBS modeling. The 10-th Inter. Conf. in Central Europe on Computer Graphics, Visualization and Computer Vision, Plenz, Czech Republic (2002) (to be appeared)

L.Piegl & W.Tiller, The NURBS Book, (2 Ed), Springer-Verlag (1997)

M.K.Hu, Visual pattern recognition by moment invariants. IRE Trans. on Information Theory, 8(2):179-187 (1962)

M.Kass, A.Witkin, D.Terzopoulos, Snakes: active contour models. Inter. J. of Computer Vision, 3(6):321-331 (1988)
P.Brigger, J.Hoeg, M.Unser, B-Spline snakes: a flexible tool for parametric contour detection. IEEE Trans. on Image Processing, 9(9):1484-1496 (2000)

R.Basri, L.Costa, D.Geiger, D.Jacobs, Determining the similarity of deformable shapes. In IEEE workshop of physics-based modeling in computer vision, pp. 135-143 (1995)

R.C.Veltkamp & M.Hagedoorn, Shape similarity measures, properties, and construction. In Advances in Visual Information Systems, Proc. of the 4th Inter. Conf., VISUAL 2000, pp. 467-476, Springer (2000)

R.Kashyap & R.Chellapa, Stochastic models for closed boundary analysis: representation and reconstruction. IEEE Trans. Information Theory, 2(5):627-637 (1981)

S.Loncaric, A survey of shape analysis techniques. Pattern Recognition, 31(8):983-1001 (1998)

S.Mallat: A theory for multiresolution signal decomposition, IEEE Trans. PAMI, 11(7):674-693 (1989)

V.N.Gudivada & J.V.Raghavan, Content-based image retrieval systems. IEEE Computer Magazine, 28(9):18-22 (1995)

W.Ma & J.P.Kruth, Mathematical modelling of free-form curves and surfaces from discrete points with NURBS. In P.J.Laurent, A. L. Mehaute & L.L.Schumaker (Ed), Curves and Surfaces in Geometric Design, A.K.Peters Ltd, Wellesley, MA, pp.319-326 (1994)

W.Niblack, R.Barber, W.Equitz, M.Flickner, E.Glasman, D.Petkovic, P.Yanker, The QBIC project: querying images by content using colour, texture and shape. In Storage and Retrieval of Image and Video Databases, SPIE, pp. 173-185 (1993)

X.Chenyang & J.L.Prince, Snakes, shapes, and gradient vector flow. IEEE Trans. on Image Processing, 7(3):359-369 (1998)

Y.Rui, A.C.She & T.S.Huang, Modified Fourier descriptors for shape representation - a practical approach. In Proc. of 1st workshop on image databases and multimedia search (1996)