A Novel And Fast Shape Description Method Using Non-Uniform Rational B-Spline (NURBS)

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Abstract: *An automated computer vision systems that resembles human capability in perceiving and understanding an image, is highly needed. This unsupervised image analysis has the ability to make decision according to the visual contents that contained in the image itself. Shape, texture and colour are the common visual contents extracted. Shape analysis has attracted a lot of attention in computer vision research due to its important role in systems for object recognition, matching and analysis. Defining the shape of an object is a difficult task, especially the computer era now has introduced the necessity to describe even very complicated shapes precisely and at the faster computation time. In this paper, we introduce a novel and fast shape description method using Non-Uniform Rational B-Spline (NURBS). Any possible form of shape is accurately described by NURBS parameters: the control points and the corresponding weights. However, one of the main difficulties in deriving a set of NURBS parameters is the determination of the optimum number of NURBS parameters to be used to model the shape of the image. Therefore, this paper presents the details of deciding the number of NURBS parameters based on the shape properties for use in the NURBS modelling process. A set of experiments is carried out to investigate the accuracy and data reduction property of the NURBS description technique using the determined number of NURBS parameters. The overall experiment results show that NURBS is an accurate shape descriptor and a potential candidate for use in the computer vision systems.*

Keywords: *shape descriptor, NURBS, number of NURBS parameters*

1. Introduction

Shape is the most prominent visual feature of an image. Perceiving a shape is to capture prominent elements of an object. Humans have the capability to determine the impression of a complete and real representation of the object. On the other hand, this is not that simple in the computer vision research. Even though, many practical shape description methods exist, there is no generally accepted methodology of shape description for use in the computer vision systems. Since the possible shape is so varied, a method for representing it must be powerful enough to capture all the salient features and has high data reduction property. The details of various shape description are found in [1].

B-Spline stands as one of the most efficient shape representation method for use in signal and image processing [2]. B-Spline is a perfect candidate for the "natural" representation of curves in which it has the following attractive properties that are suitable for shape representation and analysis:

- It represents free form shape with remarkably little data and well defined in the mathematical form.
- It has local controllability, which implies that local changes in shape are confined to the NURBS parameters local to that change.
- It has the ability to control smoothness and curvature continuity.
- It posses the characteristic of shape invariance under affine transformation, which means that the affine transformed curve is still a NURBS curve whose control points and weights are related to the original curve control points and weights through this transformation.

In this paper, we introduce Non-Uniform Rational B-Spline (NURBS), as a better spline representation compared to B-Spline. The choice of NURBS as a shape descriptor, not only offers a common mathematical form for representing free-form shapes but also geometric shapes that has sharp corners. The difference between NURBS and B-Spline is that it includes a non-uniform knot vector and an additional parameter, which is the weight. Inclusion of weight as an additional parameter adds an extra degree of freedom to NURBS and facilitates the representation of a wide variety of shapes. Furthermore, the use of non-uniform knot vectors allows better shape control and modeling of a much larger class of shapes than the uniform knot vector used in B-Spline. With these additional parameters, NURBS allows a higher compact representation, which effectively reduces the original number of the boundary points required to represent the image. Therefore, we strongly believe that NURBS is an efficient and effective shape descriptor.

In our previous work, a method for defining NURBS parameters from the boundary of the shape is presented [3]. In spite of the accuracy of the NURBS representation is very high, the NURBS modeling process still suffers from a slow computation time because our previous method applied the iterative method for finding the optimum number of NURBS parameters. A fast computation time of the shape description approach is a crucial issue in the shape analysis research. Therefore, we presented a new technique in deriving a general function for determining the number of NURBS parameters based on the shape properties extracted from the image itself. The extracted shape properties resemble the complexity of the shape, in which the number of NURBS parameters needed in the NURBS modeling process is related with the complexity of the shape. In our work, the number of corner points and the number of curvature points are the shape properties extracted in order to determine the general function.

This paper is organized as follows. Section 2 addresses the NURBS definition. In Section 3, an overview of the proposed method is presented. Section 4 introduces the NURBS modeling process and techniques to decide the optimum number of NURBS parameters. Section 5 presents the experiments to evaluate the suitability of each shape properties in deriving a general function to determine the number of NURBS parameters for use in the NURBS modeling process and reports the results. Section 6 concludes the technique presented in this paper and discusses the future directions of our research.

2. NURBS Definition

A *pth*-degree NURBS curve $C(u)$ defines a point that traces a trajectory in 2D space as the scalar parameter value u varies within the range [0, 1].

$$C(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u) w_i B_i}{\sum_{i=0}^{n} N_{i,p}(u) w_i} \tag{1}$$

where a set of n control points $\{B_i\}$ forms a control polygon and $\{w_i\}$ is the weights. An increase in the weight w_i pulls the curve closer to the control point B_i. $N_{i,p}(u)$ is the *ith* B-spline basis function of degree p (order $p+1$), defined recursively as

$$N_{i,p}(u) = \begin{cases} 1 & \text{if } u_i \le u \le u_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u) \tag{2}$$

A non–uniform knot vector, which is a nondecreasing sequence of real numbers is defined as

$$U = \left\{ \underbrace{a, \cdots a}_{p+1}, \cdots, u_{m-p-1}, \underbrace{b, \cdots b}_{p+1} \right\} \tag{3}$$

where $0 \le u_i \le u_{i+1} \le 1$, $i = 0, \cdots, m-1$ and m = number of knots. Knots in a NURBS curve are the points in parameter space where rational polynomial curves are grafted together to form a multi segment curve. Detailed explanations of NURBS can be found in [4].

3. Proposed Method

Shape description using NURBS involves determining the NURBS parameters. The NURBS parameters are the control points and weights. This procedure is referred to as NURBS modeling process and is a very challenging task. In our work, silhouette images are chosen for use in the shape description implementation. Figure 1 illustrates the overall view of the NURBS modeling process proposed in our work.

In Figure 1, the boundary points are extracted from the silhouette image using a simple boundary tracking algorithm [5]. We believe the segmentation problem of extracting an object of interest from the image itself is generally a separate problem and will not be considered to be part of the shape description problem. In our previous work, the number of NURBS parameters used to describe the image is iteratively increased from a desired initial value until the represented shape matches the original image. However, this iterative method consumes high computation time. Therefore, in this paper, we propose a new simple and direct technique based on the shape properties. As the number of NURBS parameters is computed, the knot vector is determined.

The knot vector determination of the knot vector from a set of boundary points involves two parameterization steps. The first step involves the parameterization of boundary points. Each of the boundary points is parameterized in the range of [0,1] by allocating a location parameter, u_i. For practical applications, there are three parameterization methods that are commonly used to assign the location parameters [6]. These are the uniform method, cumulative chord length method and centripetal model method. In our approach, we choose the centripetal model method because the extracted boundary points in our samples are more or less evenly spaced.

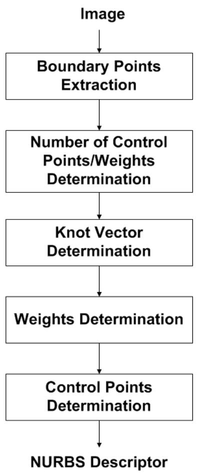

Figure 1: The general overview of the NURBS modeling process showing the step by step procedures for NURBS parameters determination.

After the boundary points are parameterized, the complete knot sequence is defined. The information of a complete knot sequence includes the spline degree, the number of NURBS parameters and the knot parameters. In this paper, the chosen spline degree is 3 in which it offers a tradeoff between the accuracy and computation time of the spline modeling. The number of NURBS parameters is obtained based on the general function that is generated based on the shape properties, as described in the following section. For the parameterization of knots, a proper knot selection method consistent with the parameterization of the boundary points is needed in order to achieve a good fitting result in the NURBS modeling process. Average Knot method is chosen because this method samples the boundary with high frequency in order to allocate more knots at places where the curve changes rapidly. The details of the parameterization methods may be found in [6].

After the knot vector is determined, a two-step linear approach for modeling the NURBS curve is introduced. During the first step, the weights are identified from a homogeneous system through Singular Value Decomposition (SVD) procedure. In SVD, an orthogonal matrix and a diagonal matrix are decomposed. A set of positive weights is generated from the combination of the eigen vectors from the orthogonal matrix. At the second step, the control points are solved with the identified weights as known parameters.

4. NURBS Modeling Process

In this paper, we address two contributions in the NURBS modeling process. First, we introduce a method for defining NURBS parameters. In this, allocation of a set of positive weights is determined according to the frequency distribution of the boundary points. This allocation method ensures a good weight fitting solution. An accurate and stable weight would ensure good control point determination. Second, we develop an efficient and effective technique in determining the optimum number of NURBS parameters. The details of determining the NURBS parameters, as well as deciding the optimum number of NURBS parameters, are discussed in the following section.

4.1 Numerical Approach: Determination of Weights and Control Points

In this approach, a homogeneous system has been derived so that the weight parameter may be computed independently. The homogeneous system defined in [7] is as follows:

$$M.w = [0]_{nx1} \tag{4}$$

where M = M_x + M_y is a n x n symmetric and non-negative matrix with:

$$M_x = N^T \overline{X}^2 N - \left(N^T \overline{X} N\right)\left(N^T N\right)^{-1}\left(N^T \overline{X} N\right)$$
$$M_y = N^T \overline{Y}^2 N - \left(N^T \overline{Y} N\right)\left(N^T N\right)^{-1}\left(N^T \overline{Y} N\right) \tag{5}$$

where

N is B-Spline basis function of degree p

$\overline{X}$ is diagonal matrix of boundary points in x-axis

$\overline{Y}$ is diagonal matrix of boundary points in y-axis

If $w \in R^n$ is a solution of Eq. 4, for all $\alpha \in R$ and $\alpha \neq 0$, αw is also a solution of Eq. 4. Owing to this property, the Eq. 4 is equivalent to Rayleigh quotient function for w, which is defined as:

$$\min_{\|w\|_2 \neq 0} R(w) = \frac{w^T Q w}{w^T w} \quad \text{where } Q = M^T M \tag{6}$$

As an application, both the general solution and the solution with positive weights of Eq. 4 are represented as a linear combination of some eigen vectors of Q corresponding to smaller eigen values. According to the properties of $R(w)$, there is an important relationship between the singular value decomposition (SVD) of M and the singular eigen value decomposition (SEVD) of Q. Owing to this relationship, one may use the SVD of M for NURBS identification [8].

In general, the weights are computed from the singular value decomposition of M, in which M is factorized as:

$$M = PDP^T \tag{7}$$

where $D = diag[d_1, d_2, \cdots, d_n]$ is a diagonal matrix whose diagonal elements are the eigen values of M in decreasing order with $d_i \geq d_{i+1} \geq 0.0$ and P is an orthogonal matrix whose columns p_i for $i = 1,2,\cdots,n$ are eigen vectors of M corresponding to d_i.

The general solution of the positive weights $w \in R^n$ with $\|w\|_2 = 0$ are given by:

$$w = \sum_{i=n-r+1}^{n} \alpha_i p_i \tag{8}$$

where r is the increment step achieved from the minimization of an objective function to obtain the best subspace of vector p, which contains positive weights, and a set of feasible solutions in this subspace. A constrained minimization algorithm is applied to the objective function to determine a set of vector α in order to find a set of best fitting solutions in this subspace. The objective function, which is derived by introducing w into Eq. 6 is:

$$\begin{cases} R(\alpha) = \min_\alpha \dfrac{\sum\limits_{j=n-r+1}^{n} \alpha_j^2 d_j^2}{\sum\limits_{j=n-r+1}^{n} \alpha_j^2} \\ subject\ to : w_l \leq w \leq w_u \end{cases} \tag{9}$$

where the value $r \in [1, n]$ will be increased until best fitting subspace can be determined and $w_u \geq w_l > 0.0$ are positive upper and lower bounds for the weights. By having a set of vector α, the weight of the corresponding control points is derived from Eq. 8.

By taking the identified weights as known parameters, the corresponding control points are obtained. When the weights are available, a non-negative least square optimization method is applied to achieve an accurate solution for control point determination. The control points in R^3 may be recovered from the control points in homogeneous space divided by the related weight.

4.2 Numerical Approach: Determination of Number of NURBS Parameter

The proposed approach is inspired by the manner in which the number of NURBS parameters needed in the NURBS modeling process is related to the complexity of the shape itself. Thus, two shape properties: number of corner points and number of curvature points are extracted from the boundary points with an intention to resemble the complexity of the shape.

In our work, the corner points detection are carried out with the boundary slitting algorithm. In this algorithm, the boundary is slitted to segments, in which the approximated corner points are detected from the boundary points in each segment having the furthest penpendicular distance from the segment. Then, the existence of the corner points are ensured if their internal angle is greater than a desired value.

The number of curvature points is determined from the boundary points that have the curvature value equal to or higher than a desired value. We used one of the curvature approximation function introduced by Williams [9] in our curvature computation of each boundary points, as derived:

$$C_i = \frac{1}{\Delta S} \sqrt{\left(\frac{\Delta P_i}{\Delta S_i} - \frac{\Delta P_{i+1}}{\Delta S_{i+1}} \right)} \tag{10}$$

where $\Delta S = \dfrac{\Delta S_i + \Delta S_{i+1}}{2}$

$$\Delta S_i = \sqrt{(P_i - P_{i-1})^2} \; ; \quad \Delta S_{i+1} = \sqrt{(P_{i+1} - P_i)^2}$$

$$\Delta P_i = \frac{(P_{i+1} - P_{i-1})}{2} \; ; \quad \Delta P_{i+1} = \frac{(P_{i+2} - P_i)}{2}$$

C_i is the curvature value of *ith* boundary point

P_i is the position of *ith* boundary point

The general function to determine the number of NURBS parameters is derived by analyzing the relationship data between the extracted shape properties value and the optimum number of NURBS parameters needed in the modeling process for a set of images. The general function is derived by approximating the relationship data by a linear approximation method. By establishing this general function, the number of NURBS parameters is easily obtained by computing the shape properties of the shape into the computed general function.

5. Experiments, Results and Discussion

In this work, we run two sets of experiments to validate the effectiveness of the proposed method in the NURBS modeling process. These have been implemented using the MATLAB software.

5.1 Experiment Methodology

The first set of experiments is carried out to derive two general functions based on the two shape properties as discussed in Section 4. In this experiment, we use 100 silhouette images as training images to obtain 100 sets of relationship data for each shape property. The optimum number of NURBS parameters for each image is generated based on the iterative method as proposed in [3]. In this method, the number of NURBS parameters is iteratively incremented until the error is equal to or less than a desired value. This error measures the dissimilarity of the reconstructed image and the original image. The desired error used is 10 pixels. The centroid-radii method is used to determine the error. In this method, both images are superimposed at their centroid point. Radii lines are projected from the centroid point to the boundary of the original image and the reconstructed image at regular interval. For each radii line, distance between the intersection points by the radii line on the boundary of the original image and the reconstructed image is computed. In these experiments, a sampling interval of 5° is used. The error is the cumulative distance between the original image and the reconstructed image at the sampled points. For each shape property, 100 relationship data are computed and a general function to determine the number of NURBS parameters is generated using the linear approximation method. In these experiments, the threshold values used for determination of corner point and curvature values are 0.95 rad and 0.4 respectively.

Based on the number of NURBS parameters computed from each general function based on the number of corner points and number of curvature points in the first experiment, as well as the iterative method, the involved NURBS modeling process are known as NCOR, NCUR and NITR respectively. Thus, the second set of experiments are aimed at proving that the NURBS parameters generated from these three NURBS modeling process have the ability to represent various shapes: geometrical shapes and free form shapes. It also validates that the generated NURBS descriptor have high data reduction and accuracy properties. The experiments are carried out on four images that include two images from the training image set and the others from non-training image set. For each reconstructed image, the data reduction property is shown by the ratio computation between the number of NURBS parameters and the boundary points. The accuracy of the reconstructed image using NURBS descriptor is shown by error that evaluated by using the centroid-radii method.

5.2 Results and Discussion

In the first set of experiments, the general function to determine the optimum number of NURBS parameters is successfully derived. Figure 2 illustrates these two general functions derived based on these two shape properties. In Figure 2(a) and 2(b), the general function is derived by approximating the relationship data based on the number of corner points and the number of curvature points.

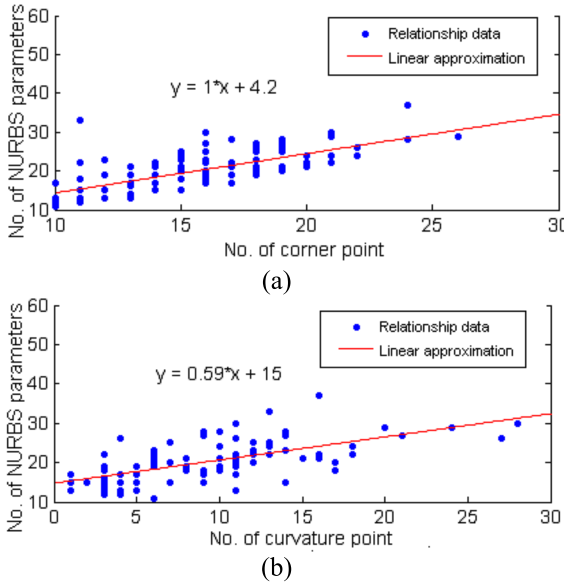

Figure 2: The general function to determine the number of NURBS parameters is derived.

In the second experiments, for each original shape, three reconstructed shape are generated from the NURBS parameters that derived from the NCOR, NCUR and NITR accordingly, as illustrated in Figure 3. The results of the experiments conducted with shapes shown in Figure 3 are presented in Table 1.

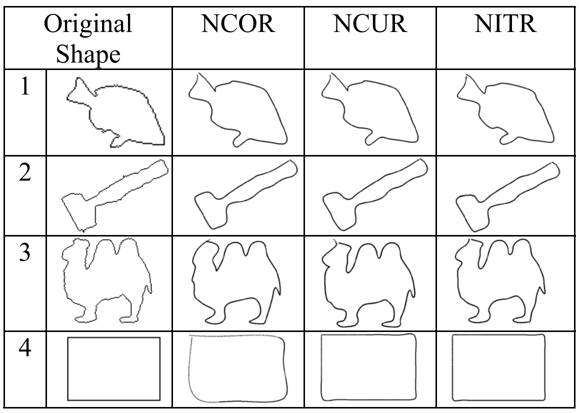

Figure 3: The original shape and the reconstructed shapes that generated from the NCOR, NCUR and NITR respectively.

In Figure 3, the results show the original shapes and the reconstructed shapes generated from the NCOR, NCOR and NITR. The reconstructed shapes of the first and second shapes are very similar to the original shapes. For the third and fourth shapes, the reconstructed shapes based on the number of curvature points are more similar to the original shapes compared to the reconstructed shapes based on the number of corner points. The results of the reconstructed shapes based on the iterative method validates that the proposed NURBS modeling process effectively generates a set of NURBS parameters to represent various shapes including the free form shape and geometrical shape, if a optimum number of NURBS parameters is provided.

Table 1: This table shows the experimental results with data reduction ratio (%), and error (pixel).

Shape	No. Boundary Points, bp	No. NURBS parameters, np	Data Reduction Ratio=bp-np/bp(%)	Error (pixel)
NCOR				
1	232	25	89.22	22
2	297	23	92.26	14
3	516	42	91.86	47
4	278	9	96.76	23
NCUR				
1	232	25	89.22	22
2	297	22	92.59	13
3	516	53	89.73	11
4	278	18	93.53	5
NITR				
1	232	33	85.78	10
2	297	27	90.91	9
3	516	60	88.37	9
4	278	24	91.37	1

It may be seen from Table 1 that the accuracy of reconstruction for first and second shapes based on the number of NURBS parameters generated from these three NURBS modeling process is very high with a maximum difference of 22 pixels. For the third and fourth shapes, the accuracy for the representation based on the NCUR is higher compared to the NCOR. This validates that the general function generated based on the curvature points is more stable and robust to determine the number of NURBS parameters, in which the difference compared to the optimum number obtained from NITR is not more than 8. For a better shape representation, the number of NURBS parameters generated from the number of curvature points may be used as an initial number to be incremented in the iterative method. Based on the overall results of the data reduction ratio between the number of NURBS parameters and boundary points, NURBS parameters generated from NCOR, NCUR and NITR is able to reduce the representation data, which is not less than 85%.

6. Conclusion and Future Work

From the results presented in Section 5, it is clear that NURBS is a powerful shape descriptor for use in computer vision systems. The derived general function is a potential method to determine the optimum number of NURBS parameters. Our future work involves exploiting this description method in representing the medical images.

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